Name: _____ Block: _____

Unit 5 – Angle Relationships

5.1 Describing, Measuring and Estimating Angles

An **angle** is formed when two rays meet at a point called the **vertex**. Angles are usually measured in degrees using a **protractor**. Angle measures range from 0° to 360°.

Angles are:

- **acute**, if their measure is between 0° and 90°

- **right**, if their measure is 90° (the two rays are perpendicular to each other)

- **obtuse**, if their measure is between 90° and 180°

- **straight**, if their measure is 180°

- **reflex**, if their measure is between 180° and 360°

In many jobs, people have to draw angles or estimate their measure. To estimate the size of an angle, you can use **referent angles,** which are angles that are easy to visualize. You can use these referents to determine the approximate size of a given angle:
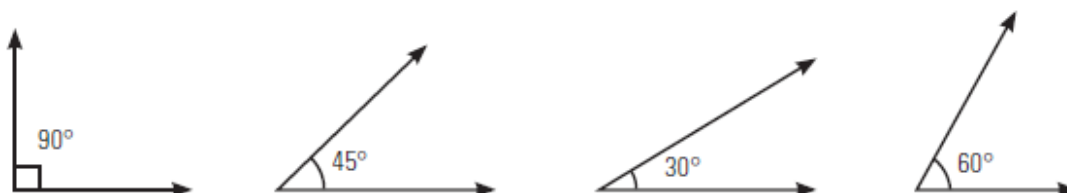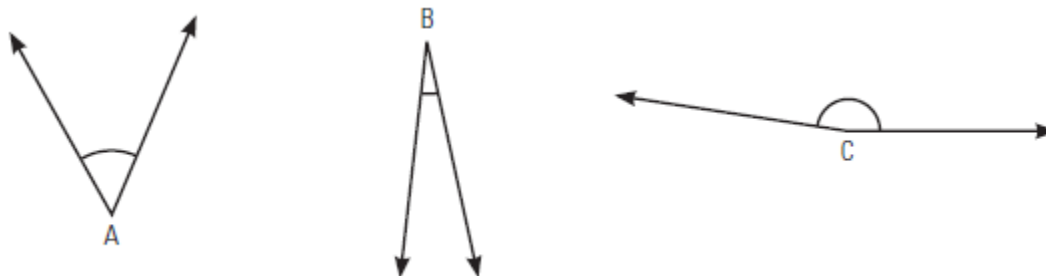

Examples

Ex 1. Identify the type of angle: acute, right, obtuse, straight, or reflex.

Workplace Math 10
Unit 5 – Angle Relationships
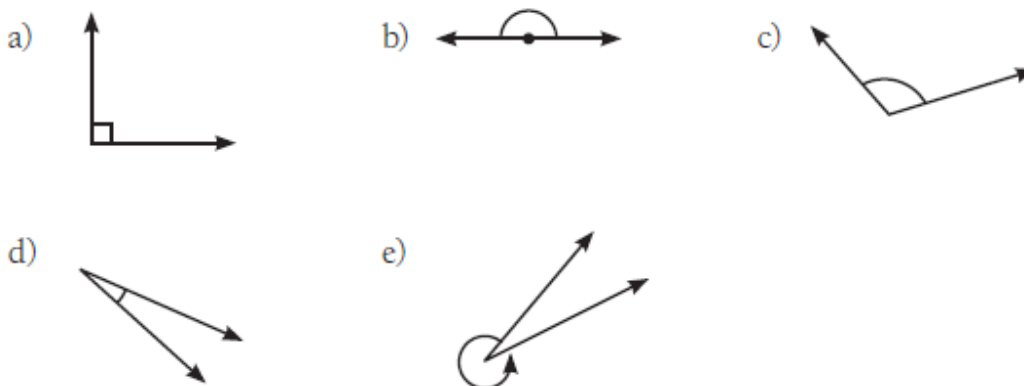
Ex 2. Measure the following angles using a protractor:

Ex 3. Use referent angles to estimate the size of each of the following angles. Then use a protractor to check your answers.
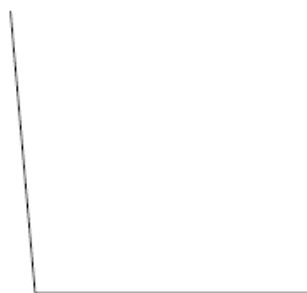
B

A

C

5.1 Practice
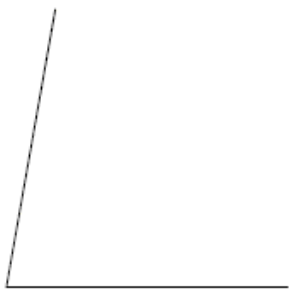
Use a protractor to measure the following angles:
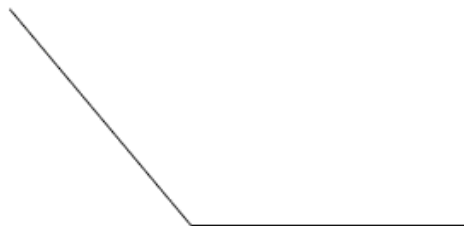
1)

2)

3)

4)

5)

6)

7. Identify the type of angle: acute, right, obtuse, straight, or reflex.

a) 68°   b) 215°   c) 91°   d) 32°

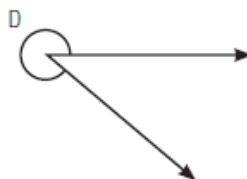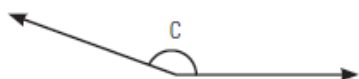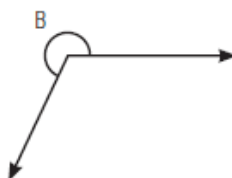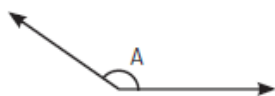e) 180°   f) 99°   g) 195°   h) 265°

8. Use referent angles to determine the approximate size of the following angles.

A   B   C   D
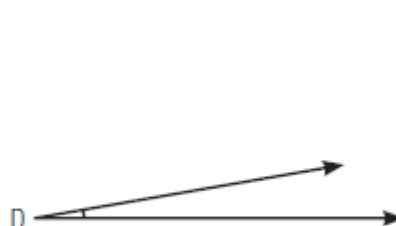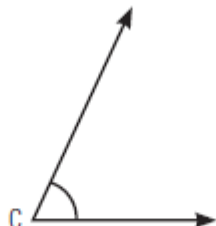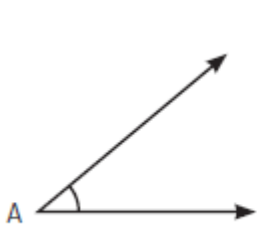
9. Use referent angles to determine the approximate size of the following angles.

A   B   C   D

3

5.2 Finding Angles Using Basic Angle Relationships – Part 1

There are five basic angle relationships that we will use as tools to find missing angles. We will start with two of these relationships.

Complementary angles: Two angles that, together, add up to 90 degrees

Supplementary angles: Two angles that, together, add up to 180 degrees

Examples

Ex 1. Determine the size of the complement and the size of the supplement of each angle – if they exist.

Angle	75°	43°	103°	87°
Complement				
Supplement				

5.2 Practice – Part 1

1. Fill in the chart with the complement and the supplement of each angle, if they exist. If they don't exist, state why.

Angle	Complement	Supplement
45°		
78°		
112°		
160°		
220°		

2. The complement of an angle is 58°.

   a) What is the size of the original angle?   b) What is the supplement of the original angle?

3. The complement of an angle is 0°.

   a) What is the size of the original angle?   b) What is the supplement of the original angle?

5.2 Finding Angles Using Basic Angle Relationships – Part 2

There are five basic angle relationships that we will use as tools to find missing angles. We have already looked at two of these in Part One.

Complementary angles: Two angles that, together, add up to 90 degrees

**Supplementary angles:** Two angles that, together, add up to 180 degrees

**Vertically opposite angles:** When two lines intersect, four angles are formed. Each opposite pair have the same angle measure.

Angles around a point: These angles will add up to 360 degrees.

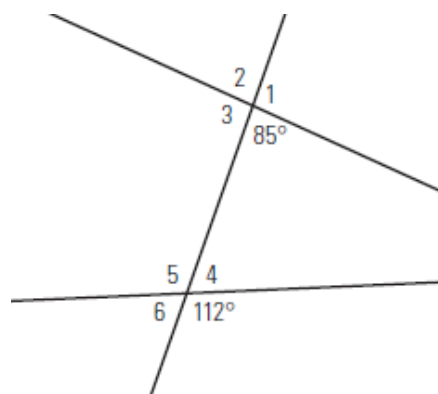**Angles in a triangle:** These angles will add up to 180 degrees.

Examples

Ex 2. Find the measure of each indicated angle. Describe the angle relationships that helped you find your answers.

a)

35°
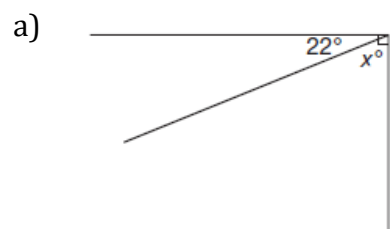
b

b)

36°

x°

c)

d)

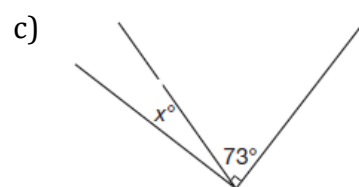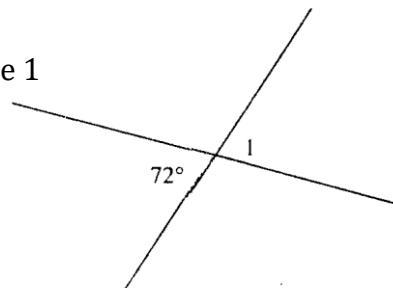e)
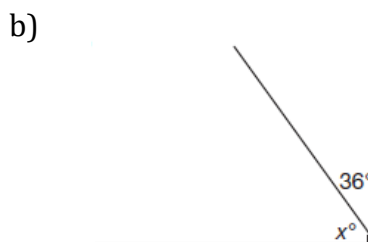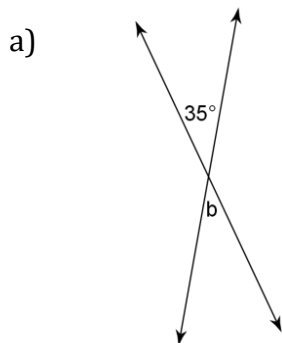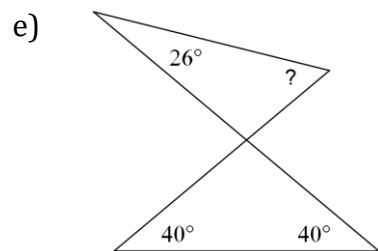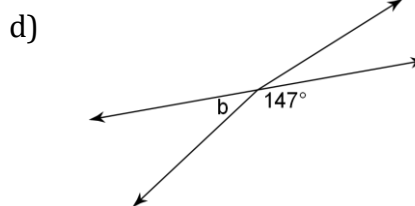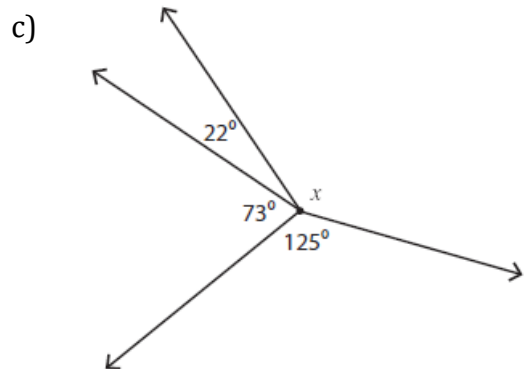
Ex 3. Find the measures of the indicated angles in the diagram.

5.2 Practice

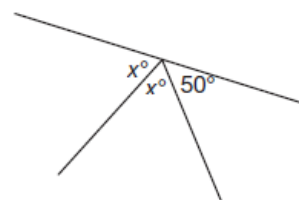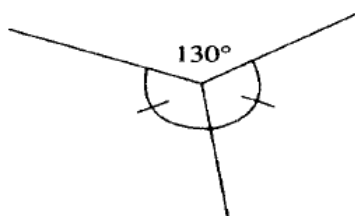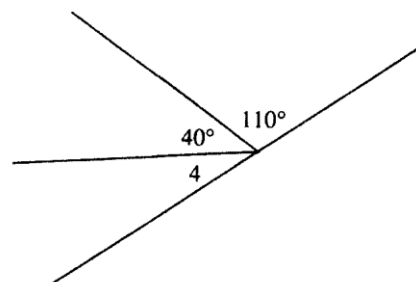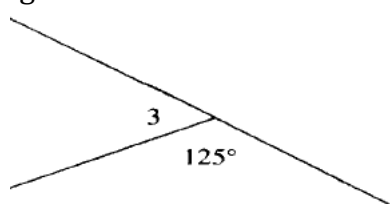4. Find the measure of each indicated angle.

a)

22° x°

b) angle 1

72°   1

c)

x°

73°

d)

x°
x° 50°

e) each missing angle

130°

f) angle 4

110°
40°
4

g) angle 3

3
125°

h) angle 5

5

i) angle 6

6
32°   116°

j) angle YZX and angle Y

Y
79°   138°
X   Z

k) angles 6 and 7

6 134°
7

l) angles 1, 2, 3 and 4

47°
2
4   3
1 65°
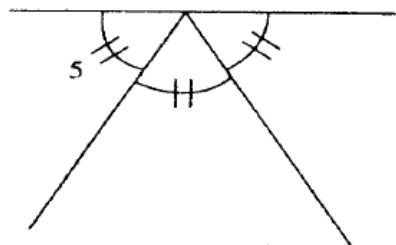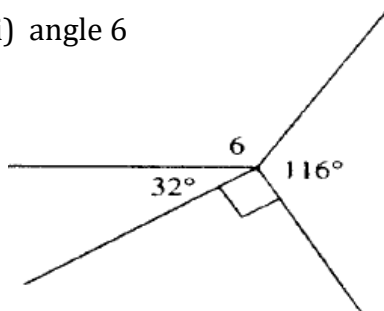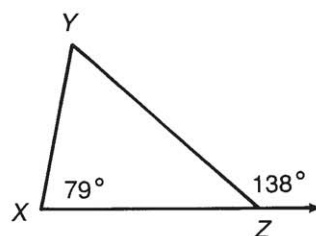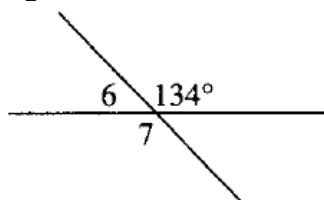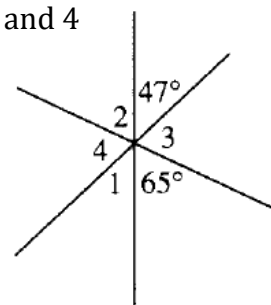
5. Find the measures of the six angles indicated in the figure.

1 120°
2 3

70° 4
6 5

9
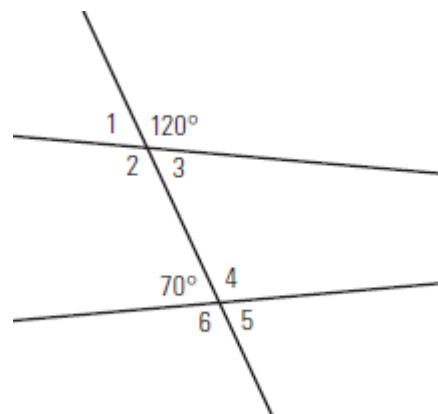
5.3 Parallel Lines, Transversals and Angle Relationships

Transversal: A line that intersects (crosses) two or more other lines

**Parallel lines:** Lines that, even when extended, will never cross each other.
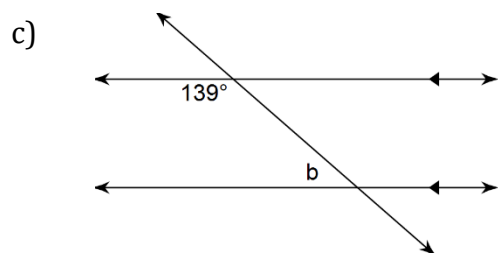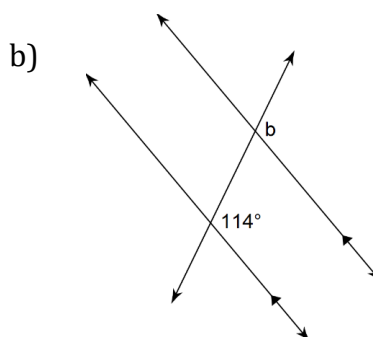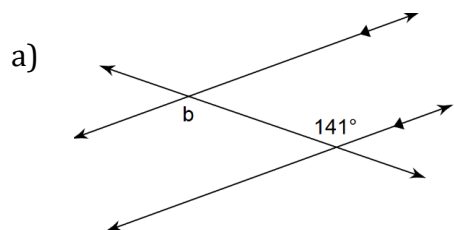
Same-side interior angles (AKA co-interior angles): Angles on the _____
of a transversal that lie _____ the parallel lines.

Alternate interior angles: Angles on _____ of a transversal that lie
_____ the parallel lines.

Corresponding angles: One _____ and one _____ angle that are
on the _____ of a transversal but not right next to each other (i.e. non-adjacent).

Examples

Ex 1. Name the relationship between the two angles, then use a protractor to find the measure of the missing angle.
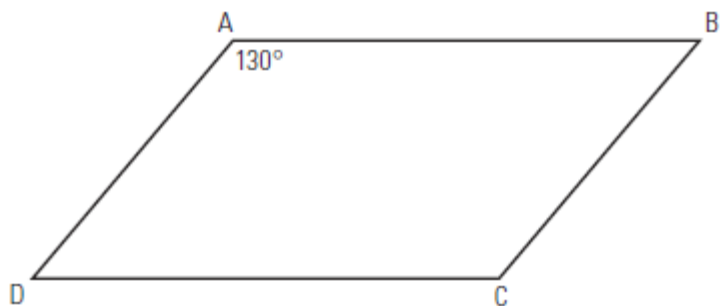
a)

141°

b

b)

b

114°

c)

139°

b

d)

93°

?

e)

?

52°

f)

?

110°
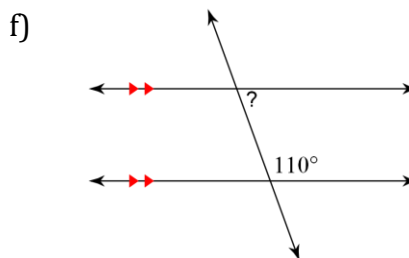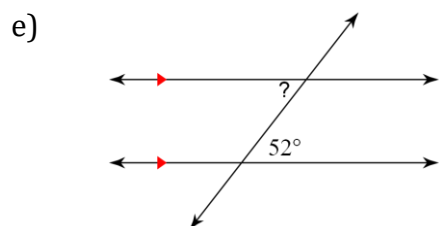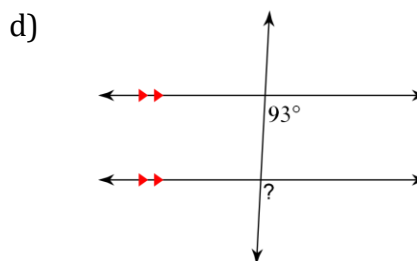
What do you notice?

Alternate interior angles are always _____.
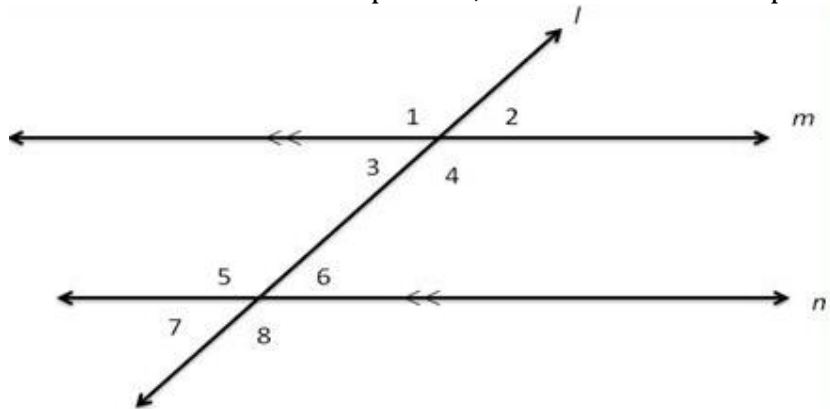
Corresponding angles are always _____.

Co-interior angles _____.

Ex 2. Given parallelogram ABCD, determine the values of each of the remaining three angles, stating your reason for each measure.

5.3 Practice

1. If lines *m* and *n* are parallel, name the relationship between the indicated pairs of angles.
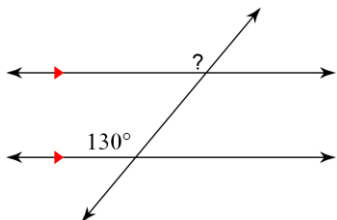

a) ∠3 and ∠6   b) ∠4 and ∠6

c) ∠2 and ∠3   d) ∠1 and ∠5

2. State the relationship between the two marked angles in each diagram, then find the measure of the indicated angle.
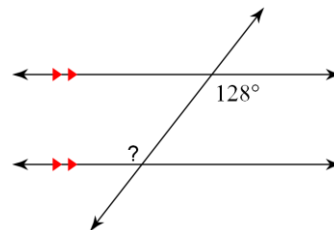
a)

Relationship:

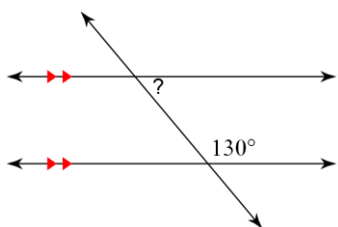Measure of indicated angle:

b)

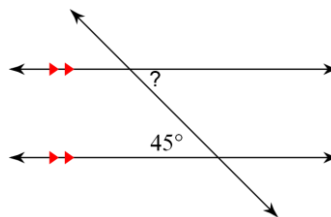Relationship:

Measure of indicated angle:

c)

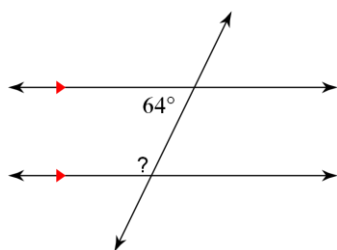Relationship:

Measure of indicated angle:

d)

Relationship:

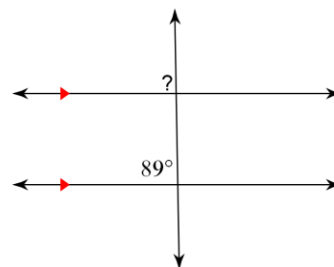Measure of indicated angle:

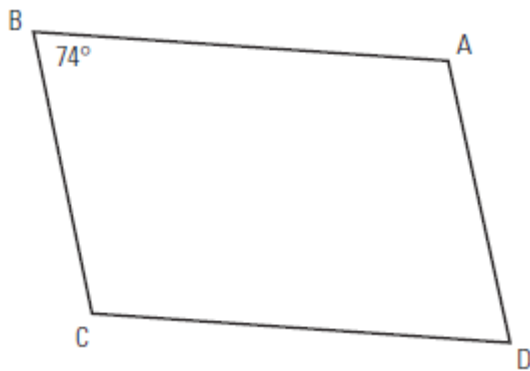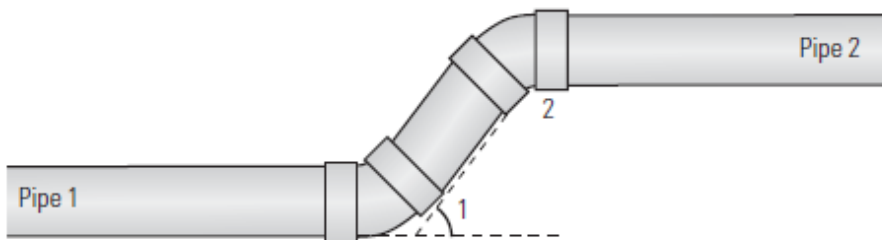e)

Relationship:

Measure of indicated angle:

f)

Relationship:

Measure of indicated angle:

3. Quadrilateral ABCD is a parallelogram. Determine the measures of the other angles and state your reasons.

4. A plumber must install pipe 2 parallel to pipe 1. He knows that ∠1 is 53°. What is the measure of ∠2? What is the relationship between these two angles?
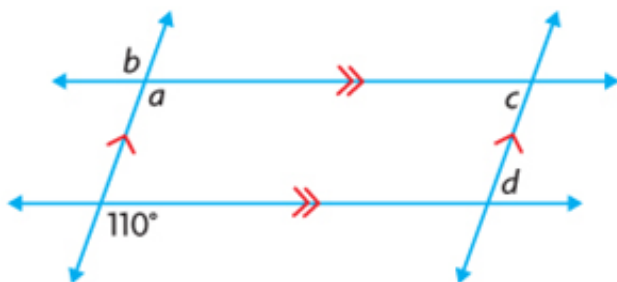
5.4 Finding Angles Involving Parallel Lines

Ex 1. In the given diagram, ℓ_1 is parallel to ℓ_2. What are the measures of the three indicated angles? Find them in order and give reasons for your answers.
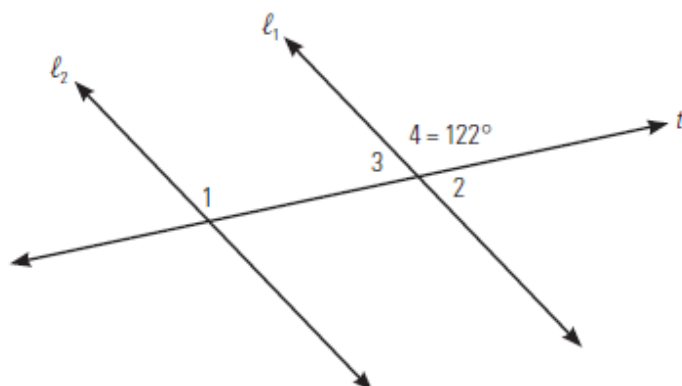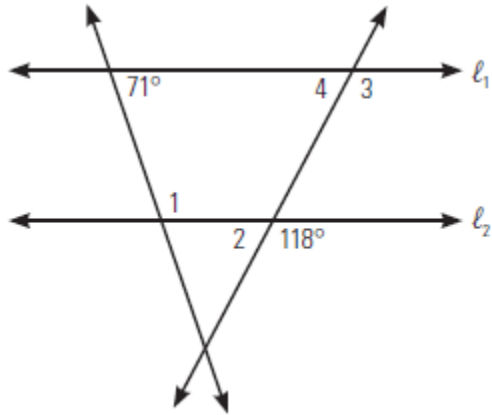

Ex 2. Determine the measures of $a, b, c,$ and d. Justify your answers.

5.4 Practice
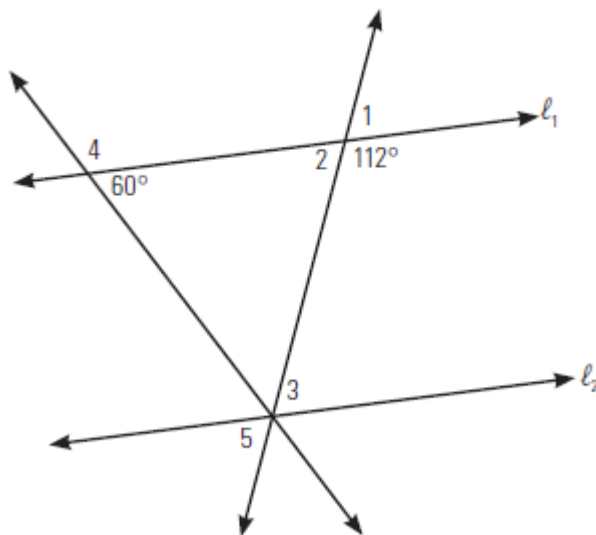
1. In the given diagram, ℓ_1 is parallel to ℓ_2. What are the measures of the three indicated angles? Give reasons for your answers.


2. In the diagram below, if the side of the house and the side of the shed are parallel, what are the measures of angles 1 and 2?
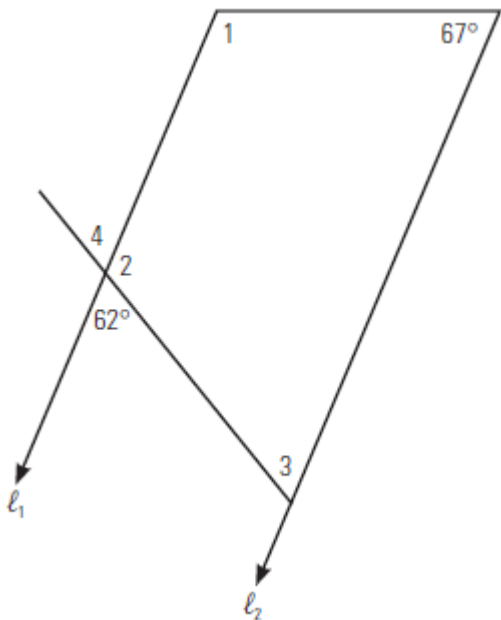
3. Given the diagram below, where ℓ_1 is parallel to ℓ_2, find the measures of the indicated angles and state your reasons.
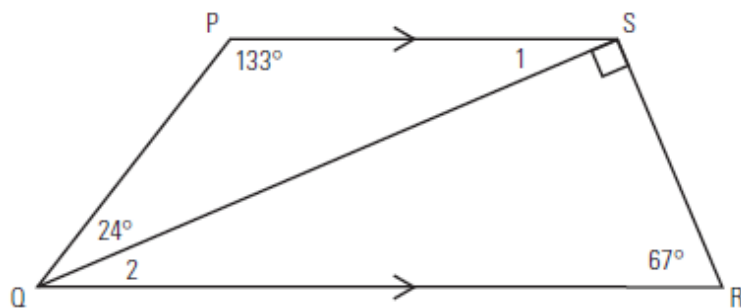
4. In the diagram below, if the top of the bridge is parallel to the deck, and the brace in the middle is perpendicular (90 degrees) to the deck, determine the sizes of angles 1 and 2.
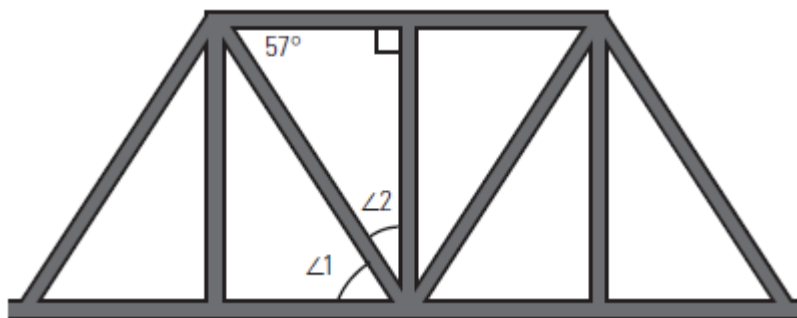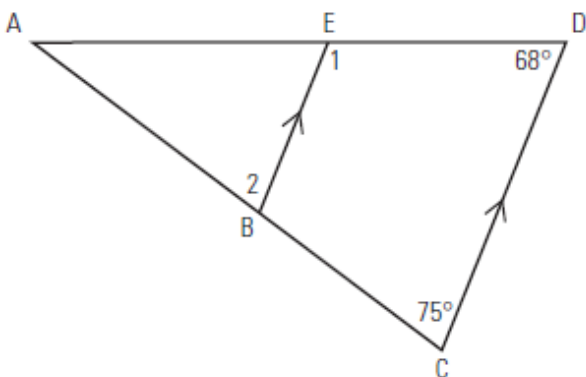
5. Given the diagram below, where ℓ_1 is parallel to ℓ_2, find the measures of the indicated angles and state your reasons.
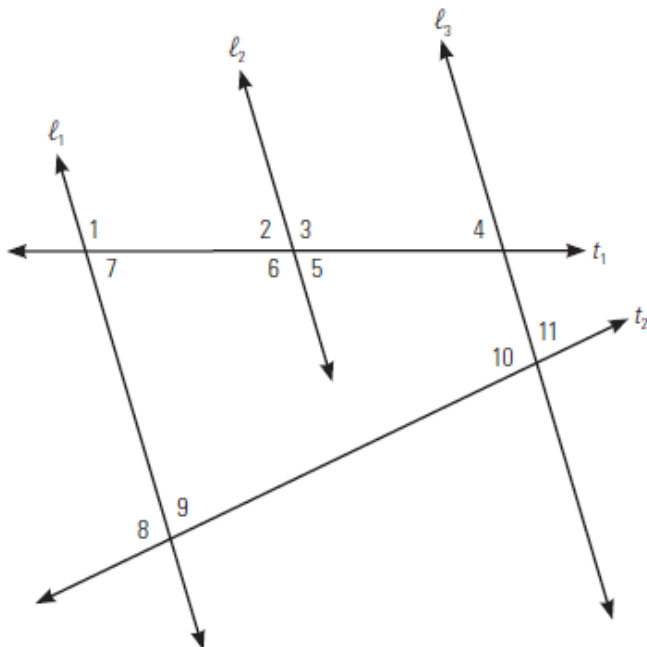
17

6. What are the measures of angles 1 and 2?
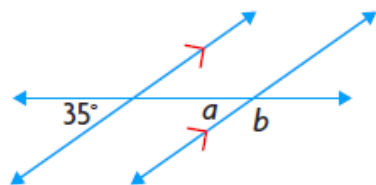
7. What are the measures of angles 1 and 2?

8. In the diagram below, ℓ_1 is parallel to ℓ_2 and ℓ_2 is parallel to ℓ_3. State two angles whose measures are the same as $\angle 7$.
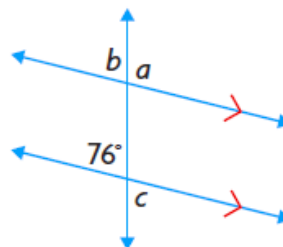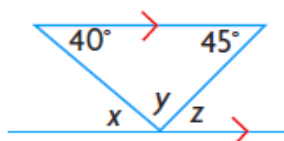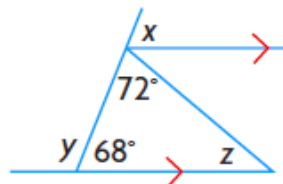
9. Determine the measures of the indicated angles.

a)

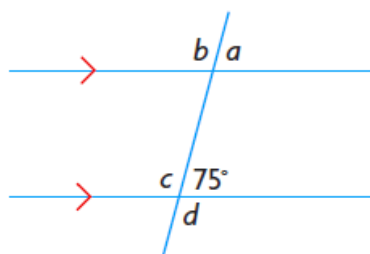35°   a b

b)

b a
76°
c

c)

40°   45°
x y z

d)

x
72°
y 68°   z

e)

b a
c 75°
d

f)

70°   x
y
120°

g)

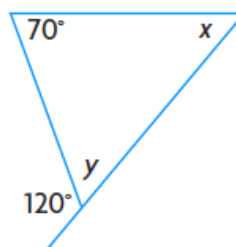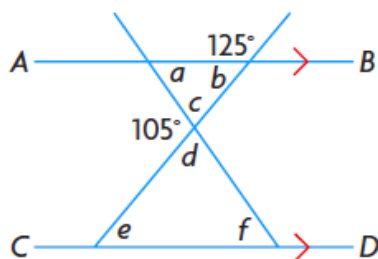A   125°   B
a b
c
105°
d
e   f
C   D

19

5.5 Determining if Two Lines are Parallel

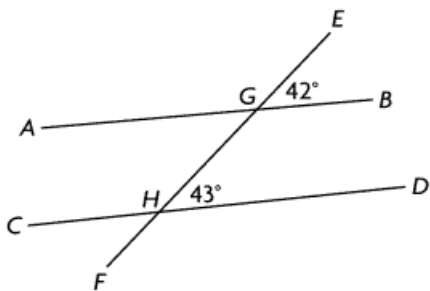If you know that, given two lines cut by a transversal:

- alternate interior angles are equal; or
- corresponding angles are equal; or
- co-interior (same-side interior) angles are supplementary;

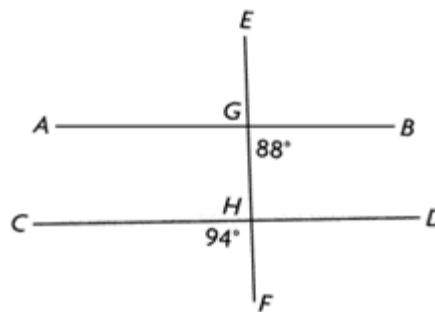then you can conclude that the lines are parallel.

Examples

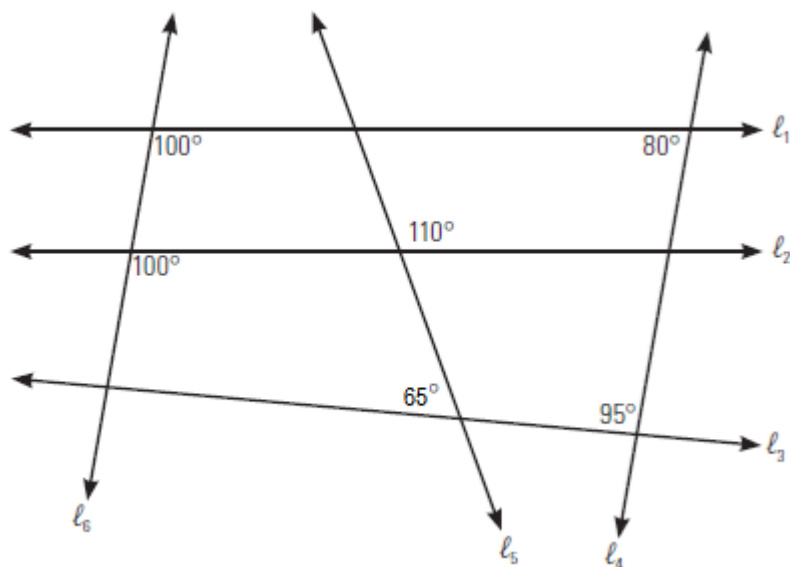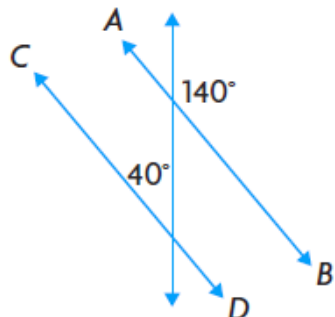**Ex 1.** In each diagram, is AB parallel to CD? Explain how you know.

a)

b)

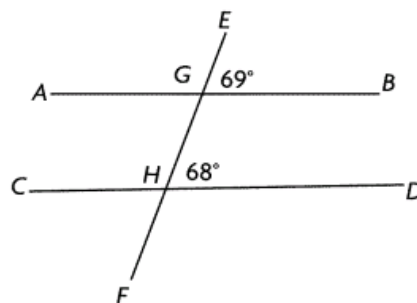**Ex 2.** Given the diagram below, prove which lines are parallel.
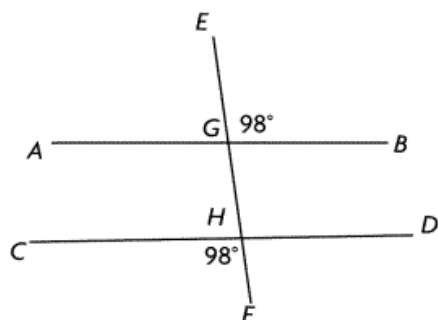
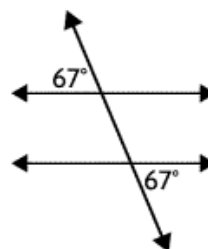## 5.5 Practice
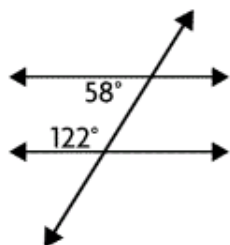
1. In each diagram, is AB parallel to CD?

a)

b)

c)

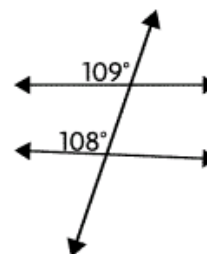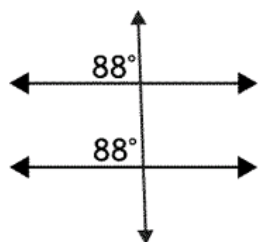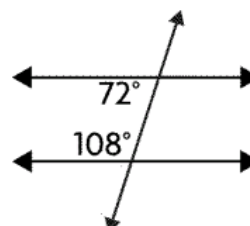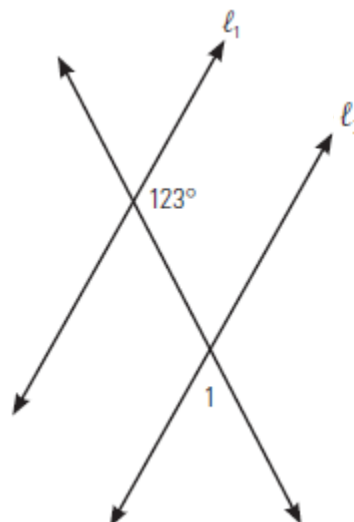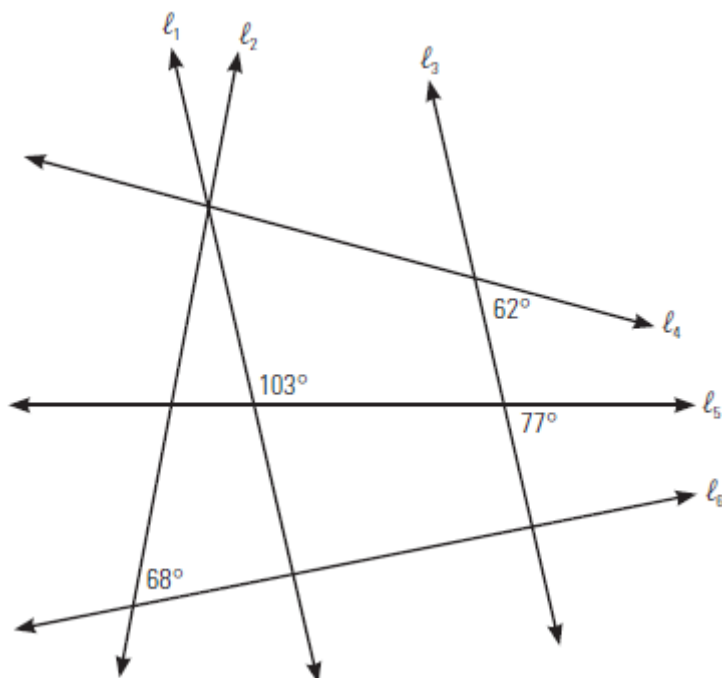d)

e)

f)

g)

h)

2. What size must angle 1 be if ℓ_1 is parallel to ℓ_2?

3. Find a pair of parallel lines in the following diagram. On the diagram, mark all the angles necessary to determine this.

ANSWERS

Section 5.1

1. 30°
2. 95°
3. 15°
4. 155°
5. 80°
6. 130°
7. a) acute
   b) reflex
   c) obtuse
   d) acute
   e) straight
   f) obtuse
   g) reflex
   h) reflex
8. A ≈ 40°, B ≈ 75°, C ≈ 65°, D ≈ 10°
   answers may vary slightly…this is okay but you should be within 10° or so
9. A ≈ 140° or 150°, B ≈ 230° or 240°,
   C ≈ 170°, D ≈ 330°

Section 5.2

1.

Complement	Supplement
45°	135°
12°	102°
None (angle is greater than 90°)	68°
None (angle is greater than 90°)	20°
None (angle is greater than 90°)	None (angle is greater than 180°)

2. a) 32° b) 148°
3. a) 90° b) 90°
4. a) 68°
   b) 72°
   c) 17°
   d) 65°
   e) 115° each
   f) 30°
   g) 55°
   h) 60°
   i) 122°
   j) ∠YZX = 42° ; ∠Y = 59°
   k) ∠6 = 46°; ∠7 = 134°
   l) ∠1 = 47°; ∠2 = 65°; ∠3 = 68°; ∠4 = 68°
5. ∠1 = 60°, ∠2 = 120°, ∠3 = 60°,
   ∠4 = 110°, ∠5 = 70°, ∠6 = 110°

Section 5.3

1. a) alternate interior angles
   b) co-interior angles
   c) vertically opposite angles
   d) corresponding angles
2. a) corresponding; 130°
   b) alternate interior; 128°
   c) co-interior; 50°
   d) alternate interior: 45°
   e) co-interior; 116°
   f) corresponding; 89°
3. ∠A = 106°; ∠C = 106°; ∠D = 74°
4. ∠2 = 127°; they are co-interior angles

Section 5.4

1. ∠1 = 109°; ∠2 = 62°; ∠3 = 118°; ∠4 = 62°
2. ∠1 = 122°; ∠2 = 90°
3. ∠1 = 68°; ∠2 = 68°; ∠3 = 68°; ∠4 = 120°;
   ∠5 = 68°
4. ∠1 = 57°; ∠2 = 33°
5. ∠1 = 113°; ∠2 = 118°; ∠3 = 62°; ∠4 = 62°
6. ∠1 = 23°; ∠2 = 23°
7. ∠1 = 112°; ∠2 = 75°
8. Any two of: ∠5, ∠2, ∠4
9. a) ∠a = 35°; ∠b = 145°
   b) ∠a = 104°; ∠b = 76°; ∠c = 76°
   c) ∠x = 40°; ∠y = 95°; ∠z = 45°
   d) ∠x = 68°; ∠y = 112°; ∠z = 40°
   e) ∠a = 75°; ∠b = 105°; ∠c = 105°; ∠d = 105°
   f) ∠x = 50°; ∠y = 60°
   g) ∠a = 50°; ∠b = 55°; ∠c = 75°; ∠d = 75°;
      ∠e = 55°; ∠f = 50°

Section 5.5

1. a) Yes
   b) No
   c) Yes
   d) Yes
   e) Yes
   f) No
   g) Yes
   h) Yes
2. ∠1 = 57°
3. ℓ_1 is parallel to ℓ_3